ACCURACY IMPROVEMENT TECHNIQUE FOR MEASURING STRESS INTENSITY FACTORS IN PHOTOELASTIC EXPERIMENT

Tae-Hyun Baek* and Christian P. Burger**

(Received October 29, 1990)

Fracture coefficients, together with the exact origin of the crack, were extracted from data sets produced an overdeterministic system solved by an iterative least squares method. Power series type williams equations were used in the analyses. The accuracy evaluation indicated that the first four terms of williams equations are sufficient to describe the stress field in the vicinity of the crack tip for both mode I and mixed mode cases. Experimental study showed that the first two terms of williams equations, which are the same as the modified westergaard equations, cannot be used to extract mixed mode fracture parameters accurately within the data collection region of $0.07 < r/a < 0.30$, where r and a are radial coordinate and crack length for an edge crack, respectively.

Key Words: Stress Intensity Factor(SIF), Crack Tip Analysis, Photoelastic Experiment, Williams Equations, Mixed Mode Fracture Parameter, Fracture Mechanics.

1. INTRODUCTION

Since the first use of the photoelastic method to study the stress field in the vicinity of an edge crack (Post, 1954), many different techniques have been tried to extract fracture parameters from the isochromatic fringes near the crack tip (Bradley and Kobayashi, 1970 ; Sanford and Dally, 1979 ; Smith, 1980). However, there has been no general agreement on which method is the most suitable for the determinations of stress intensity factors (SIFs) for cracks (Murthy and Rao, 1984).

To improve the accuracy of photoelastic data analysis for a crack tip, fringe sharpening technique (Baek, Koerner and Burger, 1988) with a digital image processing system was used. The ambiguity of the origin of a crack tip was solved by including the origin of the tip as two more unknowns, i.e., x_o and y_o as shown in Fig. 2, in the overdeterministic iterative least squares method (Sanford, 1980) by which the coefficients for the specified equations are calculated to fit the equations to the observed fringes. Power series type Williams equations (Williams, 1957) were used in the analyses. The accuracy of the experimental results were evaluated qualitatively and quantitatively.

2. PROCEDURE OF DATA ANALYSIS

2.1 Equations in the Analysis

For a homogeneous and isotropic solid in a state of plane stress or strain with zero body forces, the stress components are represented in terms of stress function, x, as

$$\sigma_r = \frac{1}{r^2}\frac{\partial^2 x}{\partial \theta^2} + \frac{1}{r}\frac{\partial x}{\partial \tau} \tag{1a}$$

$$\sigma_\theta = \frac{\partial^2 x}{\partial r^2} \tag{1b}$$

$$\tau_{r\theta} = -\frac{1}{r}\frac{\partial^2 x}{\partial \tau \partial \theta} \tag{1c}$$

For a crack ($a = \pm\pi$) shown in Fig. 1, Williams proposed the following series type of stress function (Williams, 1957).

$$x = \sum_{n=1,3,5\cdots} r^{n/2+1}\Big[A_n\Big\{\cos\Big(\frac{n}{2}-1\Big)\theta - \Big(\frac{n-2}{n+2}\Big)\cos\Big(\frac{n}{2}+1\Big)\theta\Big\} + B_n\Big\{\sin\Big(\frac{n}{2}-1\Big)\theta$$

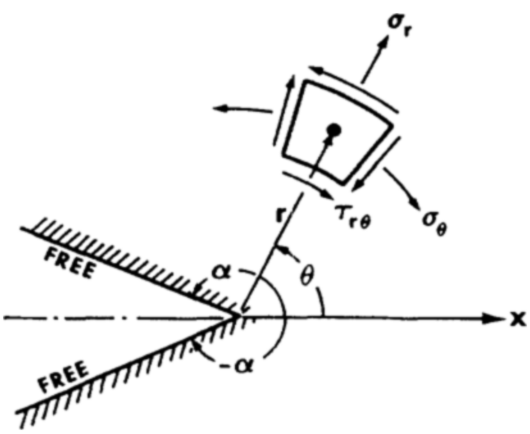


Fig. 1 Polar stress components in a sharp angular corner.

* Department of Mechanical Design, Kunsan National University, Chonbuk 573-360, Korea

** Department of Mechanical Engineering, Texas A & M University, College Station, Texas 77843, U.S.A.

$$-\sin\left(\frac{n}{2}+1\right)\theta\Big\}\Big]+\sum_{n=2,4,6\cdots} r^{n/2+1}\Big[A_n\Big\{\cos\left(\frac{n}{2}-1\right)\theta-\cos\left(\frac{n}{2}+1\right)\theta\Big\}+B_n\Big\{\sin\left(\frac{n}{2}-1\right)\theta-\left(\frac{n-2}{n+2}\right)\sin\left(\frac{n}{2}+1\right)\theta\Big\}\Big] \quad (2)$$

Note that the terms multiplied by A_n are symmetric(mode I) and the terms with B_n are anti-symmetric(mode II) with respect to $\theta=0°$. From the definition of K-factors and Eq. (1-a, b, c) and (2), mode I and mode II SIFs(K_I and K_{II}) can be defined by

$$K_I=\lim_{\substack{r\to 0\\ \theta\to 0}}\sqrt{2\pi r}\ \sigma_\theta=\sqrt{2\pi}A_1 \quad (3a)$$

$$K_{II}=\lim_{\substack{r\to 0\\ \theta\to 0}}\sqrt{2\pi r}\ \tau_{r\theta}=\sqrt{2\pi}B_1 \quad (3b)$$

2.2 Method of Photoelastic Data Analysis

The stress optic law in photoelasticity relates the isochromatic fringe order(N) to in-plane maximum shear stress (τ_{max}), which can be expressed in terms of polar stress components. Arranging the above relations, one can obtain an arbitrary funotion, G, whose value should be zero in the ideal case.

$$G(A_n,\ B_n)=\left(\frac{\sigma_r-\sigma_\theta}{2}\right)^2+\left(\tau_{r\theta}\right)^2-\left(\frac{Nf_\sigma}{2h}\right)^2 \quad (4)$$

Where f_σ and h are the material fringe value and the length of the light path in the model, respectively. The least squares fitting technique(Sanford, 1980) to calculate the unknown location of the crack tip started with the initial measured location of tip, O_m, which is assumed to deviate x_o and y_o from the corrected tip, O_c, in Fig. 2. Radius r_m and angle θ_m are the measured coordinates of a data point P at which fringe order is N. Then, the corrected corrdinates for point P are r_c and θ_c, where

$$r_c=\sqrt{(x_o+r_m\cos\theta_m)^2+(y_o+r_m\sin\theta_m)^2} \quad (5)$$

$$\theta_c=\tan^{-1}\left(\frac{y_o+r_m\sin\theta_m}{x_o+r_m\cos\theta_m}\right) \quad (6)$$

The unknown parameters, x_o and y_o, are added to the fracture coefficients A_n and B_n, and Eq.(4) can be rewritten as

$$G_k\ (A_n,\ B_n,\ x_o,\ y_o)=0 \quad (7)$$

Where the subscript k refers to the number of arbitrary data points whose number should be more than that of the unknown parameters.

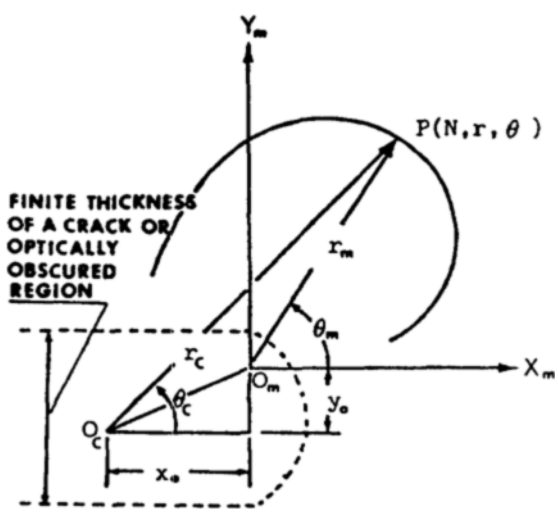


Fig. 2 Relation between measured and corrected coordinates with consideration of crack tip deviation from the initial estimated crack tip origin.

2.3 Back-Plot and Statistical Evaluation

The basic relation between isochromatic fringe order(N) and in-plane polar stress components can be expressed as

$$\left(\frac{Nf_\sigma}{2h}\right)^2=\left(\frac{\sigma_r-\sigma_\theta}{2}\right)^2+(\tau_{r\theta})^2 \quad (8)$$

Substituting Eq.(1 a,b,c) and (2) with the calculated values for coefficients A_n and B_n in place yields a polynomial type of equation in terms of $\sqrt{r}$. For a chosen angle θ and fringe order N, the corrdinate r along the line θ, where fringe order will occur, can be expressed as

$$f(\sqrt{r})=C_1(\sqrt{r})^m+C_2(\sqrt{r})^{m-1}+\cdots+C_{m+1}=0 \quad (9)$$

Where m is the order of $\sqrt{r}$ and C_1, C_2, $\cdots$, C_{m+1} are the constants of the polynomial. The polynomial can be solved efficiently by Newton's method with synthetic division, known as Horner's method(Johnson and Riess, 1982). These results are used to generate theoretical fringe loops or back-plots. When these back-plots are visually compared to the actual fringe loops, one obtain a qualitative assessement of the acouracies of the values of SIFs.

A quantitative check for the quality of fit between real and regenerated fringes is then made by simple type of statistical parameter, such as standard deviation(SD) of percentage error. For a predetermined point, the experimentally observed fringe value(N_{exp}) is known. Regenerated fringe value (N_{reg}) is also calculated at the same point. Then, the percentage error(E) between the regenerated and experimentally observed fringe value at any point is

$$E=\frac{N_{reg}-N_{exp}}{N_{exp}}\times 100\% \qquad (10)$$

For k data points, standard deviation of the percentage error can be calculated from

$$SD=\sqrt{\frac{1}{k-1}[\Sigma E_k^2-\frac{1}{k}(\Sigma E_k)^2]} \qquad (11)$$

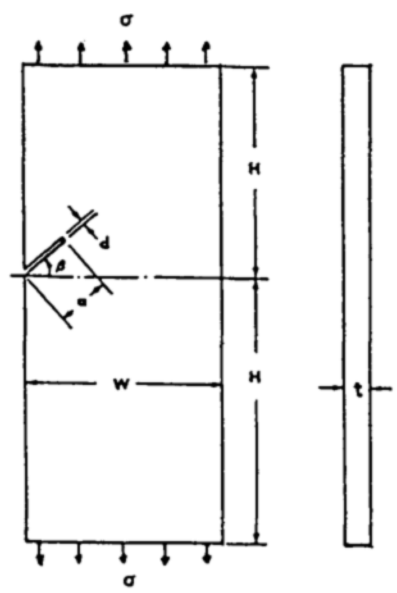
2.4 Computational Procedure

To calculate fracture parameters from the given isochromatic fringes in the neighborhood of a crack tip, and evaluate the accuracy of SIF results obtained from the analysis, programs (Baek, 1986 ; Baek, Koerner and Burger, 1988) of fringe sharpening, data acquistion, calculation of coefficients, back-plot and accuracy evaluation were developed and used.

3. EXPERIMENTS

3.1 Model Preparation

For the study of mixed mode fracture parameters, plates were cast, pre-cured and machined to the shape and size



DESIGNED GEOMETRIES

β= 0°, 22.5°, 45° d = 0.15 mm
a = 9.53 mm
W = 47.63 mm a/W = 0.2
H = 95.25 mm H/W = 2.0
t = 3.18 mm

Fig. 3 Model geometries of inclined through-thickness edge crack plate.

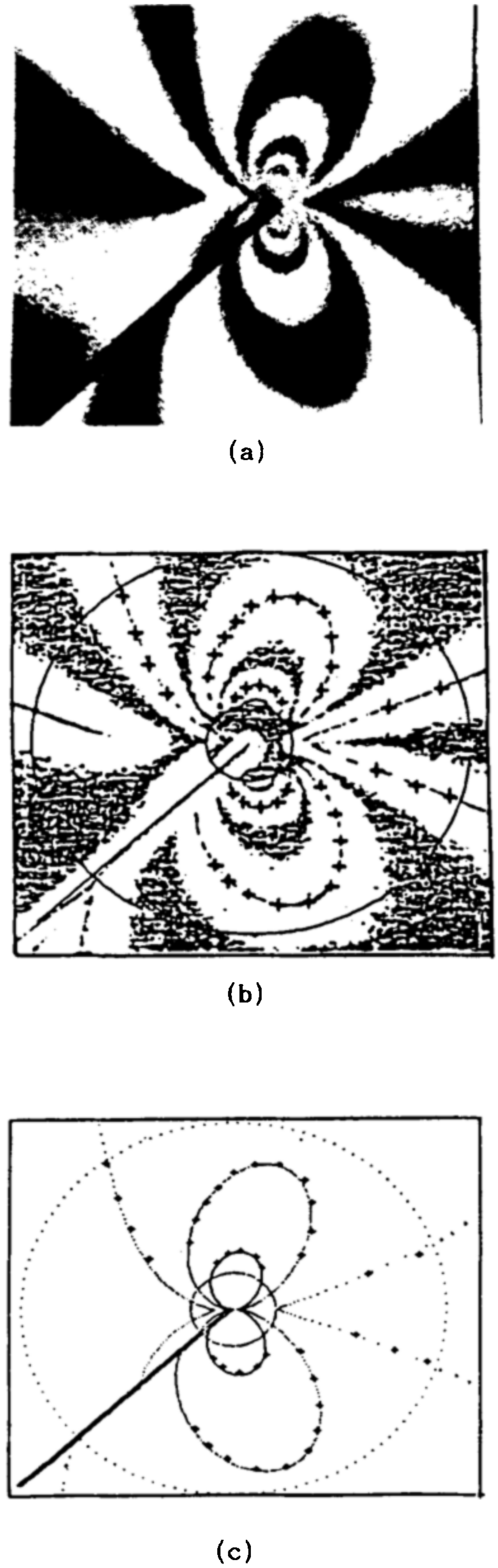
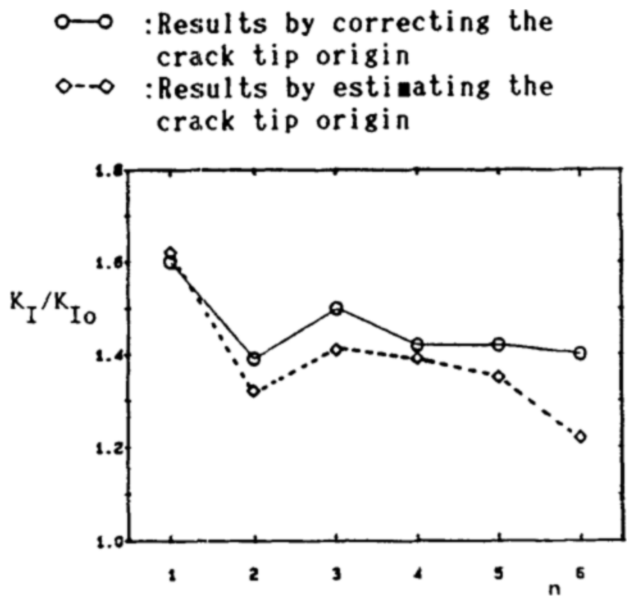
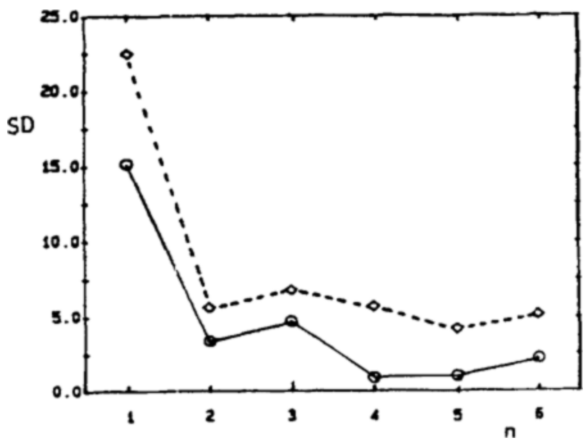


Fig. 4 (a) Isochromatic fringe loops in the vicinity of the crack tip (β=44.5°).
(b) Fringe sharpened image and collected data locations.
(c) Best back-plots drawn by using the results obtained by the first four terms of Williams equations.

shown in Fig. 3. In this study, the effect of the far boundary on the crack tip was kept small by holding $a/W=0.2$. A uniform far field tensile stress across the plate width, W, was achieved by choosing $H/W=2$. These dimensions were based on the previous research on boundary effects (Theocaris, 1972). The photoelastic material used in the experimnts were "3DMU−050" which is a product of Stress and Strain Laboratory, Dallas in Texas.



(a) K_I/K_{Io}



(b) Standard deviation (SD)

Fig. 5 Variations of SIF and statistical parameter with respect to the number of terms of Williams equations.

Before testing any of the specimens, they were loaded and viewed in white light in a circular polariscope. The uniformity of the far-field stress was checked visually to ensure uniform colar across the cross section far from the crack tip. The material fringe value of each plate was calculated from the circular disk cut from the same plate. Fig. 4(a) shows the fringe patterns around the crack tip. This image was processed by the fringe sharpening program and data were collected on the fringe sharpened lines as shown in Fig. 4(b). Fig. (c) shows the best back-plot whose standard deviation value is minimum.

2.2 Results and Discussions

For the SIF analysis from raw data collected on the fringe sharpened images, two sets of programs were developed. The first set calculates the coefficients directly from fringe data and estimated crack tip origin by the usual iterative least squares method (Sanford, 1980). The second set includes the statistical correction for the crack tip origin (Baek, 1986). Fig. 5(a), (b) show the variations of K_I/K_{Io} (where $K_{Io}=\sigma\sqrt{\pi a}$) and SD with respect to the number of terms of Williams equations when using these two sets of programs. The statistical improvement caused by an increase in the number of terms is obvious from Fig. 6. Standard deviations continuously decrease until 4 terms of the equations are used. For $n>4$, the SDs increase for all three models. SD values were minimum when 4 terms were used.

Table 1 shows the final results for two-dimensional edge cracked models. All the results for three models were obtained by using the four terms of Williams equations.

4. CONCLUSIONS

From the results presented in the preceding experiments, the following significant conclusions can be drawn.

Table 1 Test conditions and final experimental results obtained from two dimensional crack analsysis.

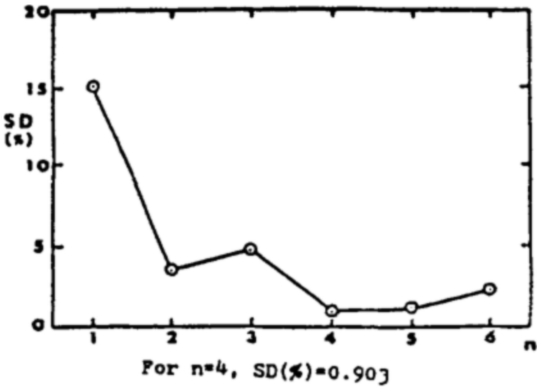
Model No.	β^a	$(a/W)_a{}^a$	σ^b (kPa)	K_I/K_{I0}	K_{II}/K_I	$\bar{E}^c$ (%)	SD^d (%)
1	−0.5°~1.5°	0.200	3524.	1.418	0.007	0.246	0.903
2	22.5°	0.200	3604.	1.289	0.232	0.111	1.303
3	44.5°	0.197	4102.	0.851	0.494	0.015	1.447

[a]See Fig. 3 for the symbols.
[b]The stress $\sigma=P/A$, where P=applied load and A=gross cross section of the model.
[c]Mean of percentage error defined by Eq. (10).
[d]Standard deviation of percentage error defined by Eq. (11).

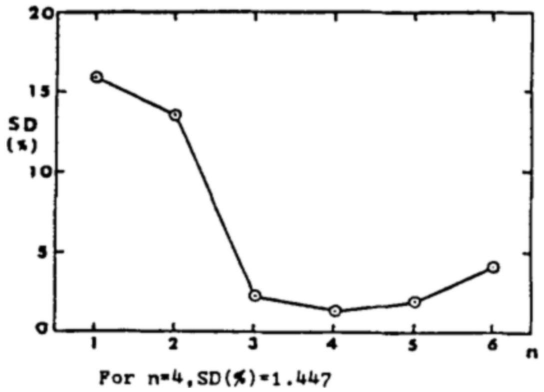
(1) Correcting the origin of the crack tip by the iterative least squares method can substantially increase the accuracy of data analysis providing that appropriate analytical equations for the stress field in the vicinity of the crack tip are used (see Fig. 5).

(2) For mixed mode cases, two terms of Williams equations, which are the same as the modified Westergoard equations (Irwin, 1958), may not be sufficient to describe the stress field around a crack tip. This is true even if the data collection region is small ($0.07 < r/a < 0.30$). Those equations can only be used to get an approximate K_I in pure mode I, and they should not be used for mixed mode analyses (see Fig. 6).

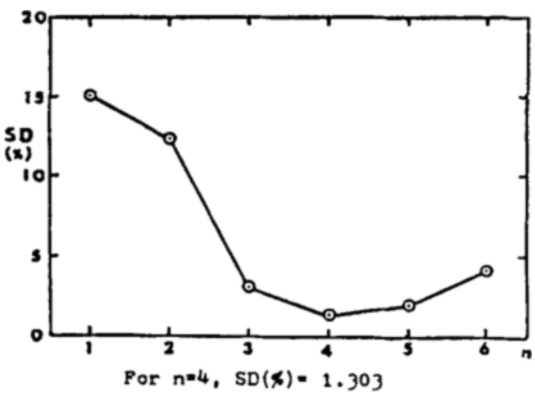
(3) Generally, for both mode I and mixed mode cases, the first four terms or $O(r)$ of Williams power series type expressions appear to be sufficient to describe the stress field arround the crack tip for the relatively close region to the crack tip used in these experiments. This conclusion can be applicable to the stress field of the crack whose interactons with other cracks or boundaries are not extreme. The inclusions of higher order terms in $r\sqrt{r}$ do not improve the accuracy of the SIF results. In fact, the quality of the results deteriorates when $n>4$ (see Fig. 6).



(a) $\beta=-0.5°\sim1.5°$



(b) $\beta=22.5°$



(c) $\beta=44.5°$

Fig. 6 Variations of standard deviation (SD) with respect to the number of terms of Williams equations.

ACKNOWLEDGEMENT

The authors acknowledge the support of National Science Foundation (Grant Number : MSM-8300376) and the Engineering Research Institute at Iowa State University, Ames, Iowa.

REFERENCES

Baek, T.H., Koerner, B.H. and Burger, C.P., 1988, "A Digital Procedure for Photoelastic Fsinge Sharpening," Proceedings of the VI International Congress on Experimental Mechanics, Vol. II, pp. 925~930.

Baek, T.H., 1986, "Study of Mixed Stress Intensity Factors by Two-and Three-Dimensional Photoelasticity," Ph. D. Dissertation, Iowa State Univrsity, Ames, Iowa.

Bradley, W.B. and Kobayashi, A.S., 1970, "An-investigation of Propagating Cracks by Dynamic Photoelasticity," Experimental Mechanics, Vol. 19, No. 3, pp.106~113.

Irwin, G.R., 1958, "Discussions of Analysis of Stress and Strains Near the end of a Crack Traversing a Plate," Proceedings of Society for Experimental Stress Analysis, Vol. 16, No. 1, pp. 93~96.

Johnson, L. W. and Riess, D. R., 1982, "Numerical Analysis," 2nd Edition, Reading, Massachusetts : Addison-Wesley Publishing Company, pp. 174~180.

Murthy, N. S. and Rao, P. R., 1984, "Photoelastic Determination of Mode I stress Intensity Factors in Tensile Strips : Effects of Crack Length," Engineering Fracture Mechanics, Vol. 20, No. 3, pp. 475~478.

Post, D., 1954, "Photoelastic Stress Analysis for an Edge Crack in a Tensile Stress Field," Proceedings of Society for Experimental Stress Analysis, Vol. 12, No. 1, pp. 99~116.

Sanford, R. J. and Dally, J. W., 1979, "A General Method for Determining Mixed Mode Stress Intensity Factors from Isochromatic Fringe Patterns," Engineering Fracture Mechanics, Vol. 11, No. 4, pp. 621~633.

Sanford, R. J., 1980, "Application of the Least Squares Method to the Photoelastic Analysis," Experimental Mechanics, Vol. 20, No. 6, pp. 192~197.

Smith, C. W., 1980, "Photoelasticity in Fracture Mechnics," Vol. 20, No. 11, pp.390~396.

Theocaris, P. S., 1972, "Interactions of Cracks with Other Cracks or Boundaries," International Journal of Fracture Mechanics, Vol. 8, No. 1, pp. 37~47.

Williams, M. L., 1957, "On the Stress Distribution at the Base of a Stationary Crack," Journal of Applied Mechanics, Vol. 24, pp. 109~114.